ABOUT THE BOOK

The title of this book is GEEPEE DRIVE, MTH 101. The book contains University 100 level Mathematics past questions and detailed solutions. The solutions are self-explanatory. The past questions are questions from Obafemi Awolowo University, Ile-Ife, Osun State, in Nigeria.

The topics covered in this book include: Set Theory, Operations with Real Numbers, Quadratic Functions and Equations, Sequences and Series, Binomial Theorem, Matrices and determinants.

The book is very useful to the fresh undergraduates in 100 level studying Engineerings and Physical Sciences in any University, Polytechnic or any higher institution of learning. It is also very useful to students who are preparing for A Level Mathematics. Also, Further Maths students of WAEC, NECO, IGCSE and other similar examinations will find the book very useful.

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CHAPTER ONE
SET THEORY

QUESTION 1

Let A, B, C be sets of a universal set U. Prove (rigorously) that

$A - (B \cup C) = (A - B) \cap (A - C)$ *(Exam question, Year ??)*

Solution:

From LHS, let $x \in \{A - (B \cup C\}$

$\Longrightarrow x \in \{A \cap (B \cup C)'\}$

$\Longrightarrow x \in A$ and $x \in (B \cup C)'$

$\Longrightarrow$

$x \in A$ and $x \notin (B \cup C)$

$\Longrightarrow$

$x \in A$ and $\{x \in B'$ and $x \in C'\}$

$\Longrightarrow (x \in A$ and $x \in B')$ and $(x \in A$ and $x \in C')$

$x \in (A \cap B')$ and $x \in (A \cap C')$

$\Longrightarrow x \in \{(A - B) \cap (A - C)\}$

Conversely, from RHS:

Let $x \in \{(A - B) \cap (A - C)\}$

$\Longrightarrow x \in (A \cap B')$ and $x \in (A \cap C')$

$\Longrightarrow (x \in A$ and $x \in B')$ and $(x \in A$ and $x \in C')$

$\Longrightarrow$

$x \in A$ and $\{x \in B'$ and $x \in C'\}$

$\Longrightarrow$

$\Longrightarrow x \in A$ and $x \notin (B \cup C)$

$\Longrightarrow x \in A$ and $x \in (B \cup C)'$

$\Longrightarrow x \in \{A \cap (B \cup C)'\}$

$\longrightarrow$

$x \in \{A - (B \cup C)\}$

$A - (B \cup C) \subseteq (A - B) \cap (A - C)$

$A - (B \cup C) = (A - B) \cap (A - C)$

QUESTION 2

On an Air Indian Flight, there are:

9 Boys --------------------- (i)

5 Indian children -------- (ii)

9 Men --------------------- (iii)

7 Foreign Boys ---------- (iv)

14 Indians ---------------- (v)

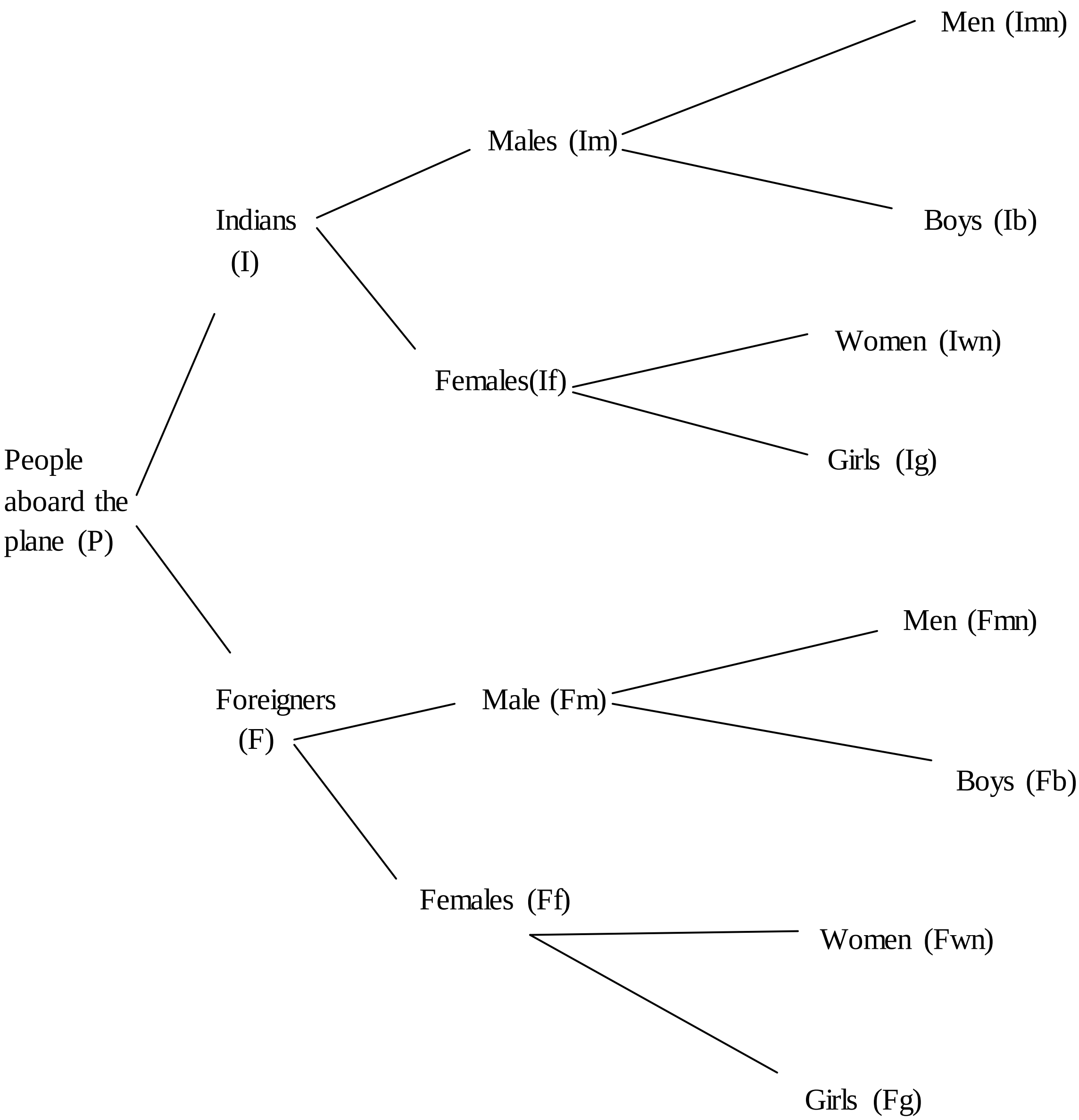
6 Indian Males and ----- (vi)

7 Foreign females------- (vii)

What is the number of people aboard the plane?

Solution:

The first thing you have to consider here is that there are basically two sets of people aboard the plane: the Indians and the Foreigners. Then, draw a tree diagram giving the analysis of the Indians and the Foreigners as shown below:



Now, considering (v) and (vi) in the data,

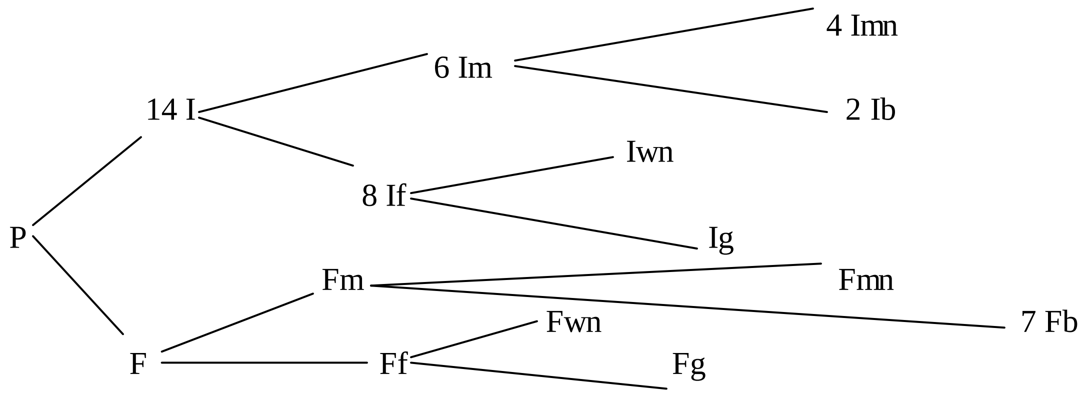
Indians = Indian males + Indian females ⟶

14 = 6 + If $\quad \therefore$ If = 14 – 6 = 8

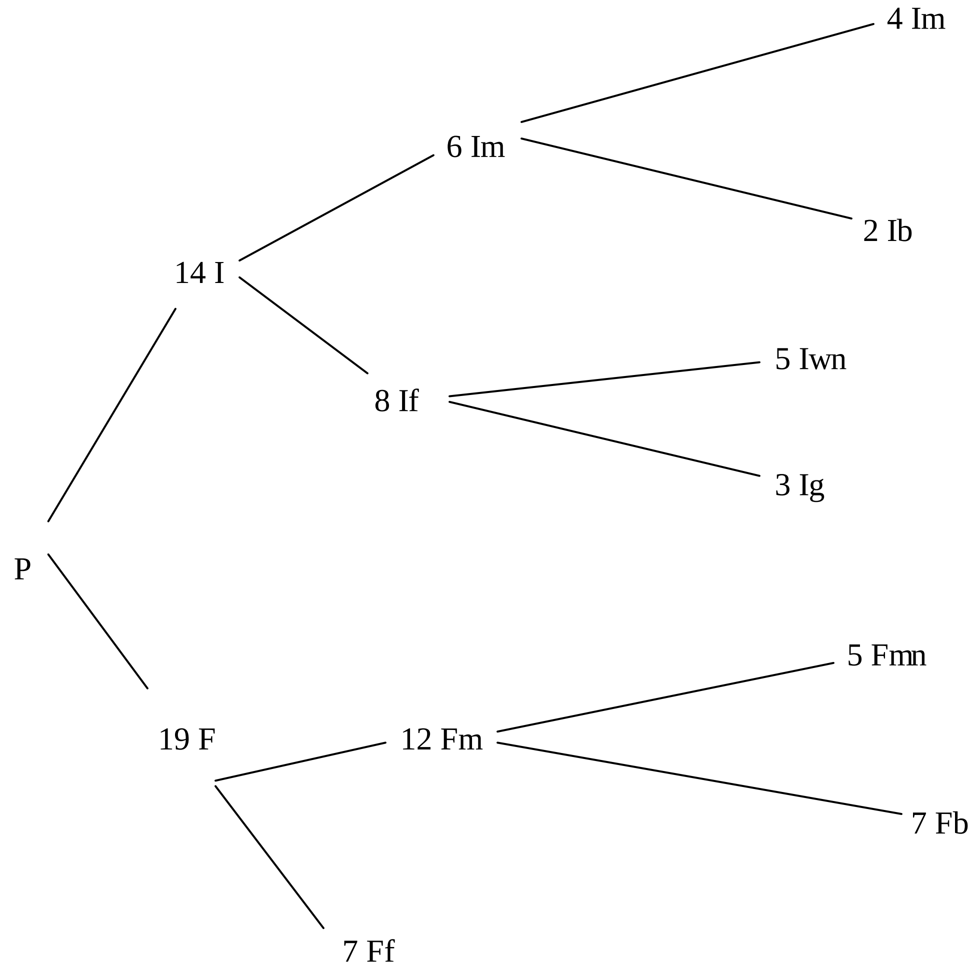
Also, considering (i) and (iv), Boys = Indian boys + Foreign boys ⟶

9 = Ib + 7 $\quad \therefore$ Ib = 9 – 7 = 2

Hence, Imn = 6 – 2 = 4 . So, we have:



Considering (ii), Indian children = Indian boys + Indian girls →
5 = 2 + Ig ∴ Ig = 5 – 2 = 3 . Hence, Iwn = 8 – 3 = 5
Considering (iii),
Men = Indian men + Foreign men → 9 = 4 + Fmn
∴ Fmn = 9 – 4 = 5. Hence, Foreign males Fm = 5 + 7 = 12 .
∴ Foreigners = Fm + Ff = 12 + 7 = 19. So we have:



Finally, the number of people aboard the plane = Indians + Foreigners = 14 + 19 = 33 .

QUESTION 3

(a) (i) If A and B are sets of a universal set U, define the Symmetric Difference Δ of A and B

(ii) Let A,B,C be sets of a universal set U. Show that:

$$A \cap (B \,\Delta\, C) = (A \cap B) \,\Delta\, (A \cap C).$$

(b) If n(A) denotes the number of elements contained in the set A, prove that

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

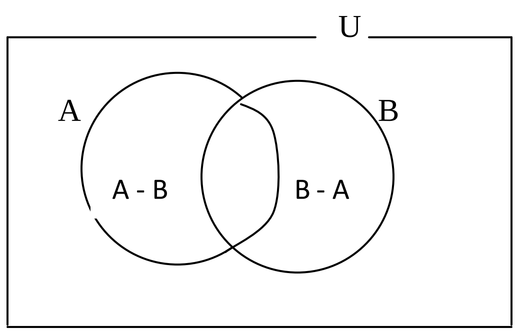
(c) Find the Cartesian product of two sets: $A = \{2, 6\}$ and $B = \{3, 7\}$ denoted by A x B. Interpret the result geometrically.

(Question 1, 1999/2000 exam)

Solution:

(a)(i) The symmetric difference of two sets A and B denoted by $A \,\Delta\, B$ is the set of element x such that x belongs to at least one of the sets $(A - B)$ and $(B - A)$.

$A \,\Delta\, B = (A - B) \cup (B - A)$ } Both are similar
$A \,\Delta\, B = (A \cup B) - (A \cup B)$



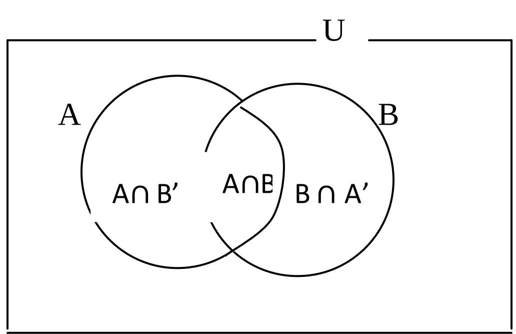
(ii) From LHS, $A \cap (B \,\Delta\, C) = A \cap \{(B - C) \cup (C - B)\}$

$= A \cap \{(B \cap C') \cup (C \cap B') = (A \cap B \cap C') \cup (A \cap C \cap B')$

$= (A \cap B - C) \cup (A \cap C - B) = (A \cap B - A \cap C) \cup (A \cap C - A \cap B)$

$= (A \cap B) \,\Delta\, (A \cap C)$

(b)



(ii)

From the diagram above,

$n(A) = n(A\cap B') + n(A\cap B) \Rightarrow n(A\cap B') = n(A) - n(A\cap B)$…….. (i)

$n(B) = n(B\cap A') + n(A\cap B) \Rightarrow n(B\cap A') = n(B) - n(A\cap B)$…….. (ii)

$n(A \cup B) = n(A\cap B') + n(A\cap B) + n(B\cap A')$……………. (iii)

Substituting (i) and (ii) in (iii) gives:

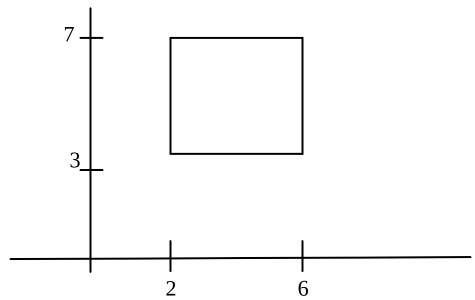
$n(A\cup B) = n(A)$ ~~$- n(A\cap B)$~~ ~~$+ n(A\cap B)$~~ $+ n(B) - n(A\cap B)$

$\therefore n(A\cup B) = n(A) + n(B) - n(A\cap B)$

(c) A = {2,6}, B = {3,7}

A x B = {(2, 3), (2, 7), (6, 3), (6, 7)}

Geometrically, it gives a square shape.



QUESTION 4

(a) For any two sets A, B of a universal set U, the operation 'x' is defined by:

$(A \times B) = \{(a, b): a \in A, b \in B\}$.

Prove rigorously that: $A \times (B\cup C) = (A \times B) \cup (A \times C)$.

(b) If n(A) denotes the number of elements contained in the set A, prove that

$n(A) + n(B) = n(A\cup B) + n(A\cap B)$

(c) 33 OAU students were asked which of the languages Spanish, French or Russian they preferred. The replies showed that 17 did not like Spanish, 18 did not like French and 20 did not like Russian. 7 students liked Spanish only, 6 students liked Russian only and 7 students liked French only. Of those who like only two of the languages, 3 did not like French. Find

(i) How many students did not like any of the languages?

(ii) How many students liked all the three languages?

Solution:

(a) From LHS, $A \times (B\cup C) = \{(a, b): a \in A, b \in (B\cup C)\}$

$\Rightarrow$ $a \in A, (b \in B$ or $b \in C)$

$\Rightarrow$ $a \in A, b \in B$ or $a \in A, b \in C$

$\Rightarrow$ $(A \times B)$ or $(A \times C)$

$\Rightarrow$ $(A \times B) \cup (A \times C)$.

Conversely, from RHS,

$(A \times B) \cup (A \times C) \Rightarrow (A \times B)$ or $(A \times C)$

$\Rightarrow$ $\{(a, b): a \in A, b \in B$ or $a \in A, b \in C)$

$\Rightarrow$ $a \in A, (b \in B$ or $b \in C)$

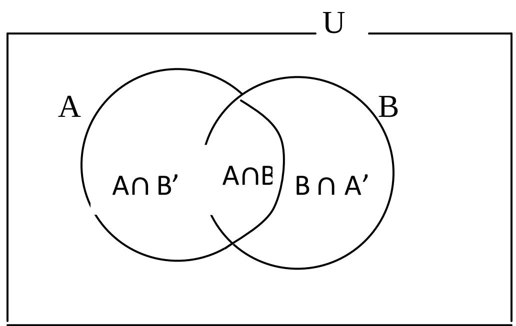
$\Rightarrow$ $a \in A, b \in (B \cup C)$

$\Rightarrow$ $A \times (B \cup C)$

$\Rightarrow$ $A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$

$\Rightarrow$ $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

(b)



From the diagram above,

$n(A) = n(A \cap B') + n(A \cap B)$ $\longrightarrow$ $n(A \cap B') = n(A) - n(A \cap B)$……… (i)

$n(B) = n(B \cap A') + n(A \cap B)$ $\longrightarrow$ $n(B \cap A') = n(B) - n(A \cap B)$………... (ii)

$n(A \cup B) = n(A \cap B') + n(A \cap B) + n(B \cap A')$…………………………………….. (iii)

Substituting (i) and (ii) in (iii):

$n(A \cup B) = n(A) - \cancel{n(A \cap B)} + \cancel{n(A \cap B)} + n(B) - n(A \cap B)$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

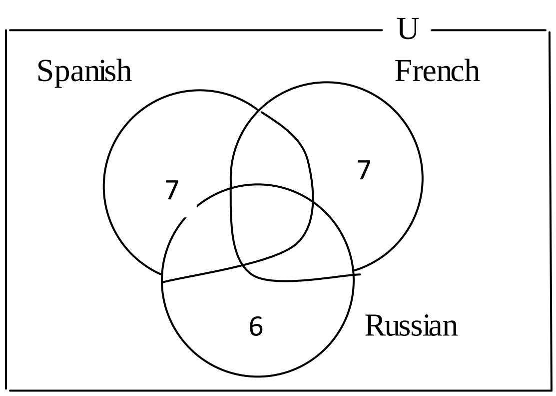
$\therefore n(A) + n(B) = n(A \cup B) + n(A \cap B)$.

(c) 7 students liked Spanish Only

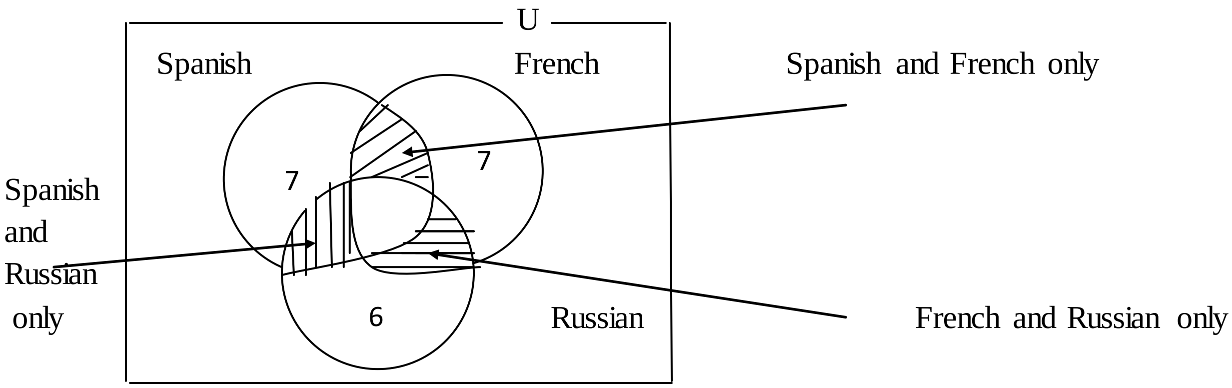
6 students liked Russian Only

7 students liked French Only

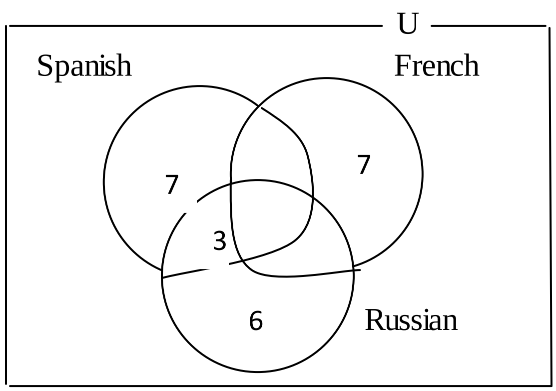
For these information we have:



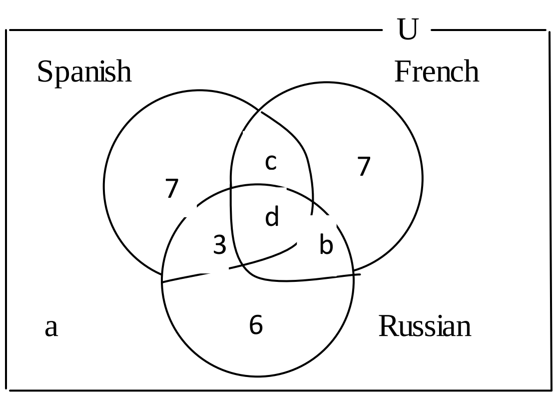
The portion of those who liked <u>only two</u> of the three languages are the shaded portions below:



Hence, from the last sentence "Of those who liked only two of the languages, 3 did not like French" We have:



The remaining information:
17 did not like Spanish ……………….. (i)
18 did not like French ………………… (ii)
20 did not like Russian ……………….. (iii)
Total OAU students asked U = 33…….. (iv)



From (ii) above, $7 + 3 + 6 + a = 18$:. $a = 2$

From (i), $7 + b + 6 + a = 17$:. $b = 2$

From (iii), $7 + c + 7 + a = 20$ $\quad \therefore\ c = 4$

From (iv), $7 + c + d + 3 + 7 + b + 6 + a = 33$ $\quad \therefore\ d = 22$

∴ (i) 2 students did not like any of the languages
(ii) 2 students liked all the three languages

QUESTION 5

(a) n(A) denotes the number of elements in the finite set A. Prove that if A and B are finite sets, then

$$n(A\cap B) + n(A\cap B) = n(A) + n(B)$$

(b) Deduce from (a) that if A, B and C are finite sets, then

$$n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(C\cap A) + n(A\cap B\cap C)$$

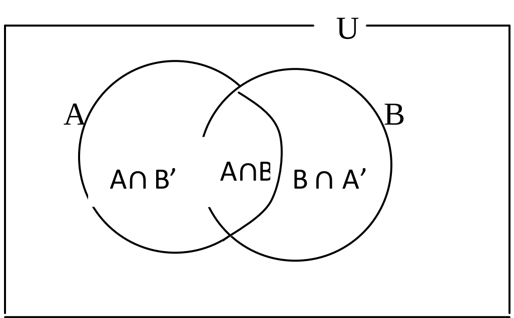
(c) A survey of 100 football fans in the France "98 World Cup provided the following data:
45 fans supported Brazil ………………………………. (i)
32 fans supported France ……………………………… (ii)
23 fans supported Nigeria …………………………..…… (iii)
19 fans supported Brazil and France………………….…. (iv)
8 fans supported France and Nigeria …………………...(v)
13 fans supported Nigeria and Brazil ………………..... (vi)
5 fans supported all supported all the three teams ……. (vii)

(i) Find the number of fans who did not support any of the three teams.
(ii) How many fans supported just one team?

(Question 6, 1997/98 Exam)

Solution:

(a)



From the diagram above,

$n(A) = n(A\cap B') + n(A\cap B)$ $\Rightarrow$ $n(A\cap B') = n(A) - n(A\cap B)$…….. (i)
$n(B) = n(B\cap A') + n(A\cap B)$ $\Rightarrow$ $n(B\cap A') = n(B) - n(A\cap B)$…….. (ii)
$n(A \cup B) = n(A\cap B') + n(A\cap B) + n(B\cap A')$……………. (iii)

Substituting (i) and (ii) in (iii) gives:

$n(A\cup B) = n(A) - n(A\cap B) + n(A\cap B) + n(B) - n(A\cap B)$
$n(A\cup B) = n(A) + n(B) - n(A\cap B)$

$\therefore \quad n(A) + n(B) = n(A \cup B) + n(A \cap B)$

(b) From the LHS, $n(A \cup B \cup C)$, let $B \cup C = D$. Then,

$n(A \cup B \cup C) = n(A \cup D) = n(A) + n(D) - n(A \cap D) = n(A) + n(B \cup C) - n\{A \cap (B \cup C)\}$

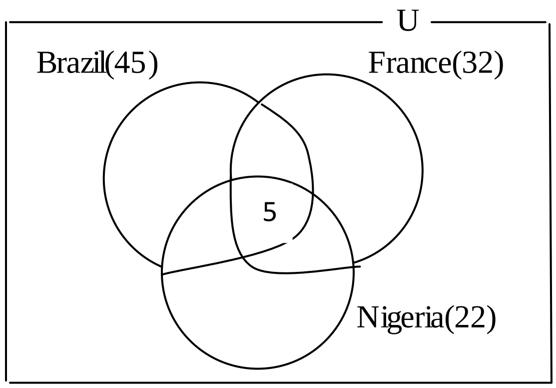
$= n(A) + n(B) + n(C) - n(B \cap C) - n\{(A \cap B) \cup (A \cap C)\}$

$= n(A) + n(B) + n(C) - n(B \cap C) - \{n(A \cap B) + n(A \cap C) - n[(A \cap B) \cap (A \cap C)]\}$

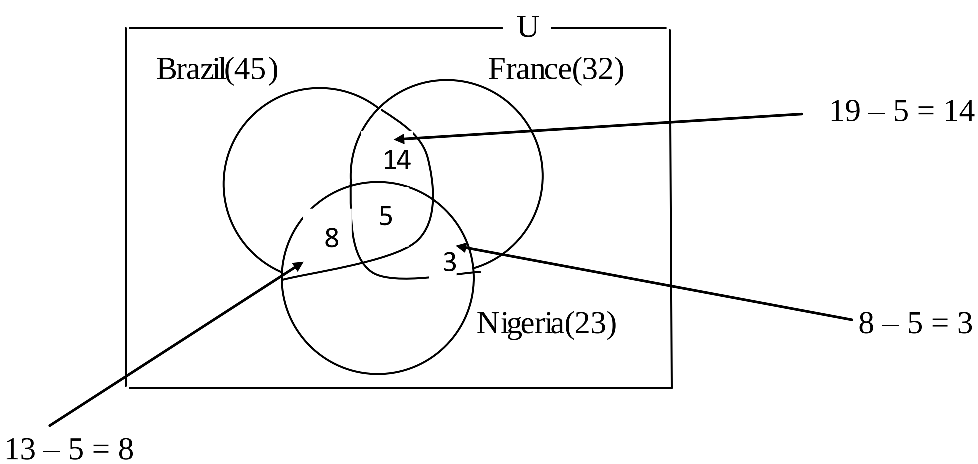
$= n(A) + n(B) + n(C) - n(B \cap C) - n(A \cap B) - n(A \cap C) + (A \cap B \cap C)$

$= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - (C \cap A) + (A \cap B \cap C)$

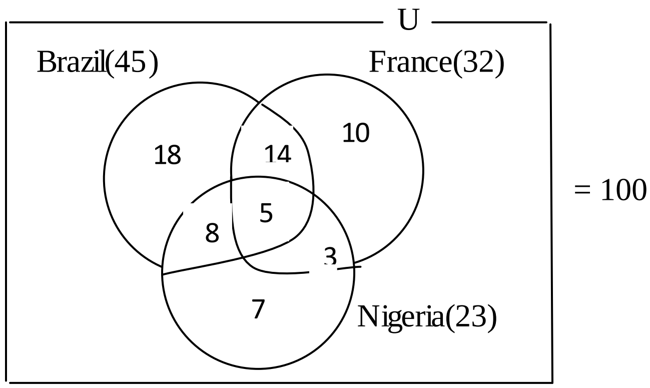
(c) From the information (i), (ii), (iii) and (vii), we have:



From (iv), (v) and (vi), we have:



Hence, we have:



The total football fans U = 100 . So,

18 + 14 + 5 + 8 + 10 + 3 + 7 + x = 100

$\Rightarrow$ 65 + x = 100 $\quad\therefore x = 100 - 65 \quad\therefore x = 35$

$\therefore$ (i) The number of fans who did not support any of the three teams = 35

(ii) The number of fans that supported just one = 18 + 10 + 7 = 35

To get the complete past questions and solutions/explanations on **Set Theory**, you can contact: 08033487161, 08177093682 or osospecial2015@yahoo.com for just N300 ($0.75).

You can also get the past questions and solutions/explanations for the remaining topics on MTH 101. The remaining topics are:

- **Operations with Real Numbers**
- **Quadratic Functions and Equations**
- **Sequences and Series**
- **Binomial Theorem**
- **Matrices and determinants.**