CHAPTER TWO
OPERATIONS WITH REAL NUMBERS

QUESTION 1

(a) Prove that for any two distinct real positive numbers a and b, $(a + b)/2 > \sqrt{ab}$.

(b) If a, b, c are distinct real numbers, show that $(a + b + c)^2 > 3(ab + bc + ca)$.

(c) Solve the following equations for real values of x, y:

$$3^{x+1} + 3^{2x} = 4$$

(d) Solve the following equations

(i) $\text{Log}(x + 1) + 2\log y = 2\log 3$…………(i)

$\text{Log } x + \log y = \log 6$…………………(ii)

(ii) $x^4 - 10x^3 + 32x^2 - 40x + 16 = 0$

(e) Find the square root of $8 - 4\sqrt{3}$.

Solution:

(a) For any two distinct real positive numbers a and b, $(\sqrt{a} - \sqrt{b})^2 > 0$.

$\Rightarrow (\sqrt{a} - \sqrt{b})(\sqrt{a} - \sqrt{b}) > 0 \quad \Rightarrow \quad a + b - 2\sqrt{ab} > 0$,

$a + b > 2\sqrt{ab} \qquad \therefore (a + b)/2 > \sqrt{ab}$.

Note that $(a + b)/2$ is the arithmetic mean of a and b while $\sqrt{ab}$ is their geometric mean.

(b) The LHS, $(a + b + c)^2 = (a + b + c)(a + b + c) =$

$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = a^2 + b^2 + c^2 + 2(ab + bc + ca)$.

But $(a + b)/2 > \sqrt{ab} \quad \Rightarrow \quad a + b > 2\sqrt{ab}$

$\Rightarrow (a + b)^2 > (2\sqrt{ab})^2 \quad \Rightarrow \quad (a + b)(a + b) > 4ab$,

$a^2 + 2ab + b^2 > 4ab \longrightarrow a^2 + b^2 > 2ab$.

If $a^2 + b^2 > 2ab$, then, $a^2 + b^2 + c^2 > ab + bc + ca$.

Hence, our expression:

$a^2 + b^2 + c^2 + 2(ab + bc + ca) > (ab + bc + ca) + 2(ab + bc + ca) \longrightarrow$

$a^2 + b^2 + c^2 + 2(ab + bc + ca) > 3(ab + bc + ca)$

$\therefore (a + b + c)^2 > 3(ab + bc + ca)$.

(c) $3^{x+1} + 3^{2x} = 4 \quad \Rightarrow$

$(3^x \text{ x } 3^1) + (3x)^2 = 4 \quad \Rightarrow \quad 3(3^x) + (3^x)^2 = 4$.

Let $p = 3^x$. So,

$3p + p^2 = 4 \quad \Rightarrow \quad p^2 + 3p - 4 = 0$,

$p^2 + 4p - 1p - 4 = 0, \quad p(p + 4) - 1(p + 4) = 0, \quad (p + 4)(p - 1) = 0$,

$\therefore p = -4$ or $1 \longrightarrow$

$3^x = -4$ ………… (i) or $3^x = 1$ ………. (ii)

For (i), there is no solution.

For (ii), $3^x = 1 \quad \Rightarrow \quad 3^x = 3^0$

$\therefore x = 0$.

(d)(i) For the first equation,

$\log(x + 1) + 2\log y = 2\log 3$, we have: $\log(x + 1) + \log y^2 = \log 3^2$

$\Rightarrow \log\{(x + 1) y^2\} = \log 9 \quad \Rightarrow \quad (x + 1) y^2 = 9$ ……….. (1)

For the second equation,

$\log x + \log y = \log 6$, $\log(xy) = \log 6 \quad \Rightarrow \quad xy = 6$…….... (2)

$x = 6/y$ ……………. (3)

Substituting (3) in (1):

$$\left(\frac{6}{y} + 1\right) y^2 = 9 \quad \Rightarrow \quad \left(\frac{6 + y}{y}\right) y^2 = 9$$

$(6 + y)\, y^2 = 9y$, $\quad 6y^2 + y^3 = 9y$,

$y(6y + y^2) = 9y$, $\quad \Rightarrow \quad y^2 + 6y - 9 = 0$. Using Quadratic Formula to find y gives:

$$y = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(-9)}}{2(1)} = \frac{-6 \pm \sqrt{36 + 54}}{2}$$

$$= \frac{-6 \pm \sqrt{90}}{2} = \frac{-6 \pm 9.49}{2}$$

$$= \frac{-6 + 9.49}{2} \quad \text{or} \quad \frac{-6 - 9.49}{2}$$

$\therefore y = 1.743$ or -7.745. So,

$$x = \frac{6}{y} = \frac{6}{1.743} \quad \text{or} \quad \frac{6}{-7.745} = 3.442 \quad \text{or} \quad -0.775$$

$\therefore (x, y) = (3.44, 1.74)$ or $(-0.76, -7.75)$

(ii) Dividing through by x^2, we have: $x^2 - 10x + 32 - \frac{40}{x} + \frac{16}{x^2} = 0$.

Collecting like terms,

$$x^2 + \frac{16}{x^2} - 10x - \frac{40}{x} + 32 = 0$$

$$\left(x^2 + \frac{16}{x^2}\right) - 10\left(x + \frac{4}{x}\right) + 32 = 0$$

Let $y = x + \frac{4}{x}$, $\quad y^2 = \left(x + \frac{4}{x}\right)^2$

$y^2 = x^2 + 16 + \frac{16}{x^2}$, $\quad y^2 - 16 = x^2 + \frac{16}{x^2}$

Our equation becomes:

$(y^2 - 16) - 10y + 32 = 0 \quad \Rightarrow \quad y^2 - 10y + 16 = 0$,

$y^2 - 8y - 2y + 16 = 0$, $\quad y(y - 8) - 2(y - 8) = 0$,

$(y - 8)(y - 2) = 0$, $\quad \Rightarrow \quad y = 8$ or 2.

So, $x + \frac{4}{x} = 8$ ……..... (i) or $x + \frac{4}{x} = 2$ ………. (ii)

For (i), $\frac{x^2 + 4}{x} = \frac{8}{1} \Rightarrow x^2 + 4 = 8x$,

$x^2 - 8x + 4 = 0$. Using Quadratic Formula to find x gives:

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{64 - 16}}{2} = \frac{8 \pm \sqrt{48}}{2}$$

$$= \frac{8 \pm 6.93}{2} = \frac{8 + 6.93}{2} \text{ or } \frac{8 - 6.93}{2}$$

.

$\therefore x = 7.46$ or 0.54

For (ii), $\frac{x^2 + 4}{x} = \frac{2}{1} \Rightarrow x^2 + 4 = 2x$, $x^2 - 2x + 4 = 0$.

Using Quadratic Formula to find x gives:

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)} = \frac{2 \pm \sqrt{4 - 16}}{2} = \frac{2 \pm \sqrt{-12}}{2}$$

$$= \frac{2 \pm \sqrt{12 \times -1}}{2} = \frac{2 \pm (\sqrt{12} \times \sqrt{-1})}{2} \quad \text{(Note: } \sqrt{-1} = i\text{)}$$

$$= \frac{2 \pm \sqrt{12}\, i}{2} = \frac{2 \pm 3.46i}{2}$$

$$= \frac{2 + 3.64i}{2} \text{ or } \frac{2 - 3.64i}{2}$$

$$= (1 + 1.73i) \text{ or } (1 - 1.73i)$$

$\therefore$ The values of x are: 7.46, 0.54, (1+1.73i) and (1–1.73i) .

Note that (1+1.73i) and (1–1.73i) are complex numbers, i.e. they are not real numbers.

(e) Let $\sqrt{8 - 4\sqrt{3}} = \sqrt{x} - \sqrt{y}$.

Squaring both sides:

$8 - 4\sqrt{3} = (\sqrt{x} - \sqrt{y})(\sqrt{x} - \sqrt{y}) \longrightarrow$

$x - 2\sqrt{xy} + y = (x + y) - 2\sqrt{xy}$

Comparing LHS with RHS,

$x + y = 8$………. (i) , $2\sqrt{xy} = 4\sqrt{3} \longrightarrow xy = 12$ …………

(ii)

Solving (i) and (ii) simultaneously:

From (ii), x = 12/y …………… (iii). Putting this into equation (i) gives:

$$\frac{12}{y} + y = 8$$

$$\frac{12 + y^2}{y} = \frac{8}{1} \longrightarrow 12 + y^2 = 8y$$

$$y^2 - 8y + 12 = 0, \qquad y^2 - 6y - 2y + 12 = 0$$
$$y(y - 6) - 2(y - 6) = 0, \qquad (y - 6)(y - 2) = 0$$
$$\therefore y = 6 \text{ or } 2$$
$$\therefore x = \frac{12}{y} = \frac{12}{6} \text{ or } \frac{12}{2} = 2 \text{ or } 6. \qquad \therefore (x, y) = (2, 6) \text{ or } (6, 2).$$

$$\therefore \sqrt{8 - 4\sqrt{3}} = (\sqrt{2} - \sqrt{6}) \text{ or } (\sqrt{6} - \sqrt{2}) = \pm(\sqrt{2} - \sqrt{6})$$

QUESTION 2:

(a) Factorise $(x^2 + 4x)^2 + 2(x^2 + 4x) - 15 = 0$.

(b) Solve for x: $(4 + \sqrt{15})^x + (4 - \sqrt{15})^x = 62$.

Solution:

(a) Let $p = x^2 + 4x \implies p^2 + 2p - 15 = 0$, $\quad p^2 + 5p - 3p - 15 = 0$

$p(p + 5) - 3(p + 5) = 0, \quad (p + 5)(p - 3) = 0$

$\therefore (x^2 + 4x + 5)(x^2 + 4x - 3) = 0$

Note: If we were asked to solve, then, $x^2 + 4x + 5 = 0$ or $x^2 + 4x - 3 = 0$

(b) To solve this type of equation involving plus (+) or minus (-) sign between the terms {i.e. $(4 + \sqrt{15})^x \pm (4 - \sqrt{15})^x = 62$), write one of the terms in brackets in form of the other:

$$\frac{1}{(4 - \sqrt{15})} \times \frac{(4 + \sqrt{15})}{(4 + \sqrt{15})} = \frac{(4 + 15)}{4^2 - (\sqrt{15})^2}$$
$$= \frac{4 + \sqrt{15}}{16 - 15} = \frac{4 + \sqrt{15}}{1} = 4 + \sqrt{15}$$

So, $\frac{1}{4 - 15} = 4 + \sqrt{15}$

The equation is now:

$$\left[\frac{1}{(4 - \sqrt{15})}\right]^x + (4 - \sqrt{15})^x = 62 \implies \frac{1}{(4 - 15)^x} + (4 - 15)^x = 62$$

Let $(4 - \sqrt{15})^x = p$.

So, $\frac{1}{p} + \frac{p}{1} = 62 \implies \frac{1 + p^2}{p} = 62$

$1 + p^2 = 62p \implies p^2 - 62p + 1 = 0$. Using Quadratic Formula to find p gives:

$$p = \frac{-(-62) \pm \sqrt{(-62)^2 - 4(1)(1)}}{2(1)}$$
$$p = \frac{62 \pm \sqrt{3844 - 4}}{} = \frac{62 \pm 61.97}{}$$

2 2

$$p = \frac{62 + 61.97}{2} \quad \text{or} \quad \frac{62 - 61.97}{2} = 62 \quad \text{or} \quad 0.015$$

So, $(4 - \sqrt{15})^x = 62$ ……… (i) or $(4 - \sqrt{15})^x = 0.015$ …………. (ii)

For (i), $\log(4 - \sqrt{15})^x = \log 62 \longrightarrow x \log(4 - \sqrt{15})^x = \log 62$

$$\therefore x = \frac{\log 62}{\log(4 - \sqrt{15})} = -2$$

For (ii), $\log(4 - \sqrt{15})^x = \log(0.015)$

$$x \log(4 - \sqrt{15}) = \log(0.015)$$

$$\therefore x = \frac{\log(0.015)}{\log(4 - \sqrt{15})} = 2$$

$\therefore$ x = -2 or 2

Note: If the sign between the terms is (x) or (+), use indices and logarithms approach directly.

QUESTION 3:

(a) Solve for x in $\sqrt{\left(\frac{a - x}{b + x}\right)} - \sqrt{\left(\frac{b + x}{a - x}\right)} = \frac{3}{2}$

(b) Resolve $\frac{(x^3 - x^2 - 3x + 5)}{(x - 1)(x^2 - 1)}$ into partial fractions.

Solution:

(a) Let $\frac{a - x}{b + x} = p$

$$\Rightarrow \sqrt{p} - \sqrt{\frac{1}{p}} = \frac{3}{2} \qquad \left(\frac{1}{\left(\frac{a - x}{b + x}\right)} = \left(\frac{b + x}{a - x}\right)\right)$$

Squaring both sides gives:

$$\left(\sqrt{p} - \sqrt{\frac{1}{p}}\right)^2 = \left(\frac{3}{2}\right)^2 \longrightarrow$$

$$p - \sqrt{1} - \sqrt{1} + \frac{1}{p} = \frac{9}{4} \longrightarrow$$

$$p - 1 - 1 + \frac{1}{p} = \frac{9}{4} \longrightarrow \quad p - 2 + \frac{1}{p} = \frac{9}{4} \longrightarrow$$

$$\frac{p}{1} - \frac{2}{1} + \frac{1}{p} = \frac{9}{4} \longrightarrow \quad \frac{p2 - 2p + p}{p} = \frac{9}{4} \longrightarrow$$

$$4(p^2 - 2p + p) = 9p \longrightarrow \quad 4p^2 - 8p + 4p = 9p \longrightarrow$$

$4p^2 - 17p + 4p = 0 \longrightarrow 4p^2 - 16p - 1p + 4p = 0 \longrightarrow$

$4p(p - 4) - 1(p - 4) = 0 \longrightarrow (4p - 1)(p - 4) = 0$

$\therefore\ p = \frac{1}{4}$ or 4

Hence,

$\frac{a-x}{b+x} = \frac{1}{4}$ or $\frac{a-x}{b+x} = 4$.

From $\frac{a-x}{b+x} = \frac{1}{4}$, $\longrightarrow 4(a - x) = 1(b + x) \longrightarrow 4a - 4x = b + x \longrightarrow$

$5x = 4a - b \quad \therefore\ x = \frac{4a-b}{5}$.

Also,

From $\frac{a-x}{b+x} = 4$, $\longrightarrow 4(b + x) = 1(a - x) \longrightarrow 4b + 4x = a - x \longrightarrow$

$5x = a - 4b \quad \therefore\ x = \frac{a-4b}{5}$.

(b) $\frac{x^3 - x^2 - 3x + 5}{(x-1)(x^2-1)} = \frac{x^3 - x^2 - 3x + 5}{x^3 - x^2 - x + 1}$ Dividing gives:

$$\begin{array}{r|l} & \quad\quad 1 \\ \hline x^3 - x^2 - x + 1 & x^3 - x^2 - 3x + 5 \\ & x^3 - x^2 - x + 1 \\ \hline & \quad\quad -2x + 4 \end{array}$$

So, $\frac{x^3 - x^2 - 3x + 5}{x^3 - x^2 - x + 1} = 1 + \frac{-2x + 4}{x^3 - x^2 - x + 1}$

Now, we resolve $\frac{-2x+4}{x^3 - x^2 - x + 1}$ into its partial fractions.

$\Rightarrow \frac{-2x+4}{(x-1)(x^2-1)} = \frac{-2x+4}{(x-1)(x-1)(x+1)}$

(Note: $x^2 - 1 = x^2 - 1^2 = (x + 1)(x - 1)$ (Difference of two squares)

Let $\frac{-2x+4}{(x-1)^2(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)}$

$\equiv \underline{A(x - 1)(x + 1) + B(x + 1) + C(x + 1)^2}$

$(x-1)^2(x+1)$

When $x = 1$,

$-2(1) + 4 = 0 + 2B + 0 \quad \Rightarrow \quad 2 = 2B \quad \therefore B = 1$

When $x = -1$

$-2(-1) + 4 = 0 + 0 + 4C \quad \Rightarrow \quad 6 = 4C \quad \therefore C = \frac{3}{2}$

$A(x-1)(x+1) + B(x+1) + C(x-1)(x-1) = Ax^2 - A + Bx + B + Cx^2 - 2Cx + C$

$= (A + C)x^2 + (B - 2C)x + (B + C - A)$

So, $-2x + 4 = (A + C)x^2 + (B - 2C)x + (B + C - A)$. Comparing coefficients gives:

$A + C = 0$ ………. (i) $\quad B - 2C = -2$ ………. (ii) $\quad B + C - A = 4$ ………. (iii)

From (i), $A + {}^3/_2 = 0 \quad \therefore A = -{}^3/_2$

Hence, $$\frac{-2x+4}{(x-1)^2(x+1)} = \frac{-{}^3/_2}{(x-1)} + \frac{1}{(x-1)^2} + \frac{{}^3/_2}{(x+1)}$$

$$= \frac{-3}{2(x-1)} + \frac{1}{(x-1)^2} + \frac{3}{2(x+1)}$$

$$\therefore \quad \frac{x^3 - x^2 - 3x + 5}{(x-1)(x^2-1)} = 1 - \frac{3}{2(x-1)} + \frac{1}{(x-1)} + \frac{3}{2(x+1)}$$

Note: In resolving a function into its partial fractions, the first thing you have to check is the relationship between the degree of numerator and the degree denominator. If the degree of numerator ≥ degree of denominator, then, you must first divide using long division as shown in the working. If the degree of numerator < degree of denominator, you should resolve the function directly.

In resolving a function into its partial fractions, there are basically four categories of functions namely:

(1) Functions with LINEAR FACTORS at the denominator

(2) Functions with NON-LINEAR FACTORS at the denominator

(3) Functions with REPEATED FACTORS at the denominator, and

(4) Functions which is an IMPROPER FRACTION

1. **Function with LINEAR FACTORS at the denominator.**

E.g. $\frac{Q}{(x-1)(2x+3)}$, $\frac{P}{(5x+1)(3x+2)(x+5)}$, etc .

In resolving this function, write:

$$\text{The function} = \frac{A}{\text{the 1st linear factor}} + \frac{B}{\text{the 2nd linear factor}}$$

For example:

$$\frac{Q}{(x-1)(2x+3)} = \frac{A}{(x-1)} + \frac{B}{(2x+3)}$$

2. **<u>Function with NON-LINEAR FACTORS at the denominator.</u>**

E.g. $\frac{Q}{(2x^2+3)(5x^3+2x^2+3x+1)}$, etc , i.e. the degree of the variable x is more than 1.

In resolving this function, write:

$$\frac{Q}{(2x^2+3)(5x^3+2x^2+3x+1)} = \frac{Ax+B}{(2x^2+3)} + \frac{Cx^2+Dx+E}{(5x^3+2x^2+3x+1)}$$

i.e. the degree of numerator should be 1 less than that of the denominator. However, before you commence this, you have to check whether the non-linear factors are factorizable into linear factors or not. This method is applicable only if the non-linear factors cannot be factorized into linear factors.

3. **<u>Function with REPEATED FACTORS at the denominator.</u>**

E.g. $\frac{Q}{(x-1)^2(2x+3)^2}$, $\frac{P}{(3x+1)^2(3x^2+4)^2}$, etc .

To resolve these, write:

E.g. $$\frac{Q}{(x-1)^2(2x+3)^2} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(2x+3)} + \frac{D}{(2x+3)^2} .$$

$$\frac{P}{(3x+1)^2(3x^2+4)^2} = \frac{A}{(3x+1)} + \frac{B}{(3x+1)^2} + \frac{Cx+D}{(3x^2+4)} + \frac{Ex+F}{(3x^2+4)^2}$$

Note that $3x^2 + 4$ is an non-linear factor.

4 **<u>Function which is an IMPROPER FRACTION</u>**

E.g. $\frac{x^3-x^2-3x+5}{x^2+2x+1}$, $\frac{x^3-x^2-3x+5}{x^3-x^2-x+1}$, $\frac{x^3-x^2-3x+5}{(x-1)(x^2-1)}$, etc .

To resolve an improper fraction, divide first using long division. Expression in question 3b above is an improper fraction.

<u>QUESTION 4:</u>

(a) Solve for x in $x^4 - 5x^3 + 6x^2 - 5x + 1 = 0$

(b) Express the rational function

$$f(x) = \frac{5 - 6x - 8x^2}{(1 + 2x)^2(1 - x)} \quad \text{in partial fractions.}$$

Solution:

(a) Dividing through by x^2 gives:

$$x^2 - 5x + 6 - \frac{5}{x} + \frac{1}{x^2} = 0$$

Collecting the like terms gives:

$$x^2 + \frac{1}{x^2} - 5x - \frac{5}{x} + 6 = 0$$

$$\left(x^2 + \frac{1}{x^2}\right) - 5\left(x + \frac{1}{x}\right) + 6 = 0$$

Let $y = x + \frac{1}{x} \Rightarrow y^2 = x^2 + \frac{1}{x^2} + 2 \longrightarrow$

$$y^2 - 2 = x^2 + \frac{1}{x^2}$$

Hence, the equation becomes:

$(y^2 - 2) - 5y + 6 = 0$

$y^2 - 5y + 4 = 0, \quad y^2 - 4y - 1y + 4 = 0$

$y(y - 4) - 1(y - 4) = 0, \quad (y - 4)(y - 1) = 0 \longrightarrow$

$y = 4$ or 1

So, $x + \frac{1}{x} = 4$ .... (i) or $x + \frac{1}{x} = 1$ .... (ii)

For (i), $\frac{x}{1} + \frac{1}{x} = \frac{4}{1} \longrightarrow$

$$\frac{x^2 + 1}{x} = \frac{4}{1} \longrightarrow x^2 + 1 = 4x \longrightarrow$$

$x^2 - 4x + 1 = 0$. Using Quadratic Formula to find x gives:

$$x = \frac{-b \pm\sqrt{b^2 - 4ac}}{2a} \longrightarrow$$

$$x = \frac{-(-4) \pm\sqrt{(-4)^2 - 4(1)(1)}}{2(1)} = \frac{4 \pm \sqrt{16.4}}{2}$$

$$x = \frac{4 \pm\sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2}$$

$$x = \frac{4 + 2\sqrt{3}}{2} \text{ or } \frac{4 - 2\sqrt{3}}{2} = 2 + \sqrt{3} \text{ or } 2 - \sqrt{3}$$

` For (ii), $x + \frac{1}{\ } = 1 \longrightarrow \frac{x^2 + 1}{\ } = 1$

x x

$x^2 + 1 = x \longrightarrow x^2 - x + 1 = 0$. So,

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)} = \frac{1 \pm \sqrt{1-4}}{2}$$

$$= \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm (\sqrt{3})i}{2}$$

$$\therefore x = \frac{1 - (\sqrt{3})i}{2} \quad \text{or} \quad \frac{1 + (\sqrt{3})i}{2}$$

: The values of x are:

$2 + \sqrt{3}$, $2 - \sqrt{3}$, $(\frac{1}{2} + \frac{\sqrt{3}}{2}i)$ and $(\frac{1}{2} - \frac{\sqrt{3}}{2}i)$.

Note:

$(\frac{1}{2} + \frac{\sqrt{3}}{2}i)$ and $(\frac{1}{2} - \frac{\sqrt{3}}{2}i)$ are complex numbers.

$\sqrt{-3} = \sqrt{3 \quad x \quad -1} = \sqrt{3}$ x $\sqrt{-1} = \sqrt{-3}$ i ($\sqrt{-1}$ is always represented by i)

(b) Let $$\frac{5 - 6x - 8x^2}{(1 + 2x)^2(1 - x)} \equiv \frac{A}{(1 + 2x)} + \frac{B}{(1 + 2x)^2} + \frac{C}{(1 - x)}$$

$$\equiv \frac{A(1 + 2x)(1 - x) + B(1 - x) + C(1 + 2x)^2}{(1 + 2x)^2(1 - x)}$$

Now, equating the numerators to each other:

$5 - 6x - 8x^2 = A(1 + 2x)(1 - x) + B(1 - x) + C(1 + 2x)^2$

When x = 1, we have:

$5 - 6(1) - 8(1)^2 = 0 + 0 + 9C \longrightarrow -9 = 9C \quad \therefore C = -1$

When x = - ½ ,

$5 - 6(-½) - 8(-½)^2 = 0 + \frac{3}{2}B + 0 \longrightarrow 6 = \frac{3}{2}B \quad \therefore B = 4$

To find A, we have to:

(i) expand the RHS numerator
(ii) express it as a polynomial of x variable and
(iii) compare its coefficient with that of LHS numerator.

So, $A(1 + 2x)(1 - x) + B(1 - x) + C(1 + 2x)^2 = A(1 + x - 2x^2) + B(1 - x) + C(1 + 4x + 4x^2) =$

$A + Ax - 2Ax^2 + B + Bx + C + 4Cx + 4Cx^2 = 4Cx^2 - 4Ax^2 + Ax - Bx + 4Cx + A + B + C =$

$(4C - 2A)x^2 + (A - B + 4C)x + (A + B + C)$

LHS numerator = $-8x^2 - 6x + 5$

So, we have: $4C - 2A = -8$ ……… (i) $A - B + 4C = -6$ ……….. (ii)
$A + B + C = 5$ ………. (iii)
Using equation (iii) with $C = -1$, $B = 4$ obtained above, we have:
$A + 4 - 1 = 5 \longrightarrow A = 2$
Therefore,

$$\frac{5 - 6x - 8x^2}{(1 + 2x)^2(1 - x)} = \frac{2}{(1 + 2x)} + \frac{4}{(1 + 2x)^2} - \frac{1}{(1 - x)}$$

Note that you can use either equation (i) or (ii) also to determine A.

QUESTION 5

(a) Express $\dfrac{4}{16r^2 + 8r - 3}$ in partial fractions.

(b)(i) Show that if $y = \log_a x$, then, $\log_x a = \frac{1}{y}$ where $a > 0$, $x > 0$.

(ii) Hence or otherwise, find the solution set of the inequality

$$\log_x a \leq \log_a x, \quad a > 1.$$