2009 COWBELL NATIONAL SECONDARY SCHOOLS MATHEMATICS COMPETITION

DATE:

MONDAY, MAY 18, 2009

TIME:

11.30 AM

VENUE:

AIRPORT HOTEL, IKEJA, LAGOS.

EXAMINATION DURATION:

1 HOUR

PART TWO

INSTRUCTIONS

1. ATTEMPT ALL QUESTIONS
2. ORDERLINESS AND CLARITY WILL ATTRACT EXTRA MARKS
3. THE USE OF CALCULATOR IS NOT ALLOWED

1. Solve completely the equations

$x + y + z = 1$

$x^2 + y^2 + z^2 = 35$

$x^3 + y^3 + z^3 = 97$ ....12 Marks

2a Determine the area of the largest rectangle that can be inscribed in the circle $x^2 + y^2 = a^2$

2b. Name the rectangle so formed ....12 Marks

3 . An electrical resistance , R ohms, of a copper wire varies directly as its length, Im , and inversely as the square of its radius, rcm. A copper wire of length 625m and radius 0.25cm has a resistance of 24 ohms.

(a) Derive an equation connecting R,I and r.
(b) Calculate the radius of the same type of copper wire of length 500m and resistance 30 ohms. .... 12 Marks

4a. Using a ruler and a pair of compasses only, Construct

i. triangle ABC, such that $|BC| = 7.5$cm $\angle ABC = 120°$ and $\angle ACB = 22.5°$ Measure AC.

ii. the locus L_1 of points equidistant from B and C.

Iii. the locus L_2 of points 5cm from B.

4b Measure $P_1 P_2$, where P_1 and P_2 are the points of intersection of L_1 and L_2 .... 14 Marks

1. If $\tan x = {}^5/_{12}$, where x is acute, find the value of $39\sin x + 26\cos x$.
   A. 17 B. 22 C. 24 D. 26 E. 39

   Use the information below to answer questions 2 and 3. If 3 boys and 2 girls sit in a row, find the probability that

2. the boys and girls each sit together
   A. 0.1 B. 0.2 C. 0.3 D. 0.4 E. 0.6

3. the girls sit together
   A. 0.2 B. 0.3 C. 0.4 D. 0.5 E. 0.6

4. Solve the equation for all positive values of θ less than $360°$.
   $3\tan\theta + 2 = -1$
   A. $135°$ or $315°$ B. $45°$ or $135°$
   C. $315°$ or $45°$ D. $315°$ or $180°$
   E. $360°$ or $315°$

5. The standard deviation of the ages of a group of children is 1.4 years. What will be the standard deviation of the ages of these same children next year?
   A. 1.4 years B. 1.5 years C. 1.6 years.
   D. 1.7 years E. 1.8 years

6. A section of metal pipe has an outer diameter of 5cm and the metal is 3mm thick. The pipe is 18cm long. What volume of metal is used in making the section of pipe?
   A. 79.7cm² B. 68.4cm² C. 63.5cm²
   D. 61.6cm² E. 59.7cm²

7. Which of the following lines is not parallel to the line $3y+2x+7=0$?
   A. $3y+2x-7=0$ B. $9y+6x+17=0$
   C. $24y+16x+19=0$ D. $3y-2x+7=0$
   E. $15y+10x-13=0$

8. Calculate along the equator the distance between two points lying on the equator and on longitudes $42°W$ and $21°E$. Take radius of the earth as 6400km
   A. 1173km   B. 2346km C. 3520km
   D. 4693km E. 7040km

9. If $x = 1$ is a root of the equation $x^3 - 2x^2 - 5x + 6$, find the other roots.
   A. -3, 2 B. -2, 2 C. 3, -2 D. 1, 3 E. -3, 1

10. If $263 + 441 = 714$, what is the number base used?
    A. 5 B. 6 C. 7 D. 8 E. 9

11. If $(x-2)$ and $(x+1)$ are factors of the expression $x^3 + px^2 + qx + 1$, what is the sum of p and q?
    A. 0   B. -3 C. 3   D. $-{}^{17}/_3$ E. $-{}^2/_3$

12. The sides of a triangle are $(x+4)$ cm, x cm and $(x-4)$cm respectively. If the cosine of the largest angle is ${}^1/_5$, find the value of x.
    A. 24cm B. 20cm C. 28cm D. ${}^{88}/_7$cm E. 0cm.

13. Find a factor which is common to all three binomial expressions $4a^2 - 9b^2$, $8a^3 + 27b^3$, $(4a+6b)^2$
    A. 4a+6b   B. 4a-6b   C. 2a+3b
    D. 2a-3b   E. None

14. In the equation below, solve for x if all the numbers are in base 2.
    $11/x = 1000/(x + 101)$.
    A. 101 B. 11 C. 110 D. 111   E. 10

15. In a class of 120 students, 18 of them scored an A grade in mathematics. If the section representing the A grade students on a pie chart has angle $x°$ at the centre of the circle, what is x?
    A. 15 B. 27 C. 50 D. 52 E. 54

16. If mustapha scored 75 in mathematics instead of 57, his average mark in four subjects would have been 60. What was his total mark?
    A. 282 B. 240 C. 222 D. 210 E. 220

17. Four boys and ten girls can cut a field in 5 hours. If the boys work at ${}^1/_4$ the rate at which the girls work, how many boys will be needed to cut the field in 3 hours?
    A. 180 B. 60 C. 25 D. 20 E. 18

18. The length of a square is increased by 20% while its width is decreased by 20% to form a rectangle. What is the ratio of the area of the rectangle to the area of the sqaure?
    A. 6:5 B. 25:24 C. 5:6 D. 24:25 E. 1:2

19. PQRS is a rhombus. If $PR^2 + QS^2 = KPQ^2$, find K.
    A. 1 B. 2 C. 3 D. 4 E. 5

20. In a regular pentagon, PQRST, PR intersects QS at O. Calculate angle ROS.
A.36° B.72° C.108° D.144° E.156°

21. Five people are to be arranged in a row for a group photograph. How many arrangements are there if a married couple in the group insist on sitting next to each other?
A.48 B.24 C.20 D.10 E.8

22. A committee consists of 12 members. A minimum quorum at meetings of this committee consists of 8 members. In how many ways can a quorum occur?
A. 640 B.686 C.774 D.794 E.842

23. If k+1, 2k-1, 3k+1 are three consecutive terms of a geometric progression, find the possible values of the common ratio.
A. 0, 8   B. -1, $^{5}/_{3}$ C. 2, 3
D. 1, -1   E. -2, $^{2}/_{3}$

24. The mean of the ages of ten students is 16, but when the age of their teacher is added to it, the mean becomes 19. Find the age of the teacher.
A.27 B.35 C.38 D.49 E.51

25. A bag contains 16 red and 20 blue balls only. How many white balls must be added to the bag so that the probability of randomly picking a red ball is eqaul to $^{2}/_{5}$
A.4   B.20 C.24 D.40 E.42

26. Mrs. dada reduced the quantity of food bought for her family by 10% when she found that the cost of living had increased by 15%. Thus the fractional increase in the family food bills is now
A.$^{1}/_{12}$ B.$^{5}/_{35}$ C.$^{19}/_{200}$ D.$^{7}/_{200}$ E.$^{5}/_{100}$

27. A ladder resting on a vertical wall makes an angle whose tangent is 2.4 with the ground. If the distance between the foot of the ladder and the wall is 50cm, what is the length of the ladder?
A. 1m B. 1.1m C. 1.2m D. 0.9m E. 1.3m

28. If x varies inversely as y, and y varies directly as the square root of z, and z varies directly as 1/w, write in words how x varies with w.
A. x varies inversely as w^2
B. x varies directly as w^2
C. x varies directly as w
D. x varies inversely w
E. x varies directly as square root of w.

29. Find a two digit number such that three times the tens digits is 2 less than twice the units digit and twice the number is 20 greater than the number obtained by reversing the digits.
A.24 B.42 C.74 D.47 E.72

30. A stone S is tied to a point P vertically above S by an elastic string of length 2m. How high does the stone rise when the string is inclined at an angle 60° to the vertical?
A.1m B.2m C.3m D.4m E.5m

31. Two distinct sectors in the same circle subtend 100° and 30° respectively at the centre of the circle. Their corresponding arcs are in ratio
A. 1:100 B. 3:1 C. 5:2 D. 10:3 E.13:30

32. The price of oranges was raised by 1/2 kobo per orange, the number of oranges a customer can buy for N2.40 will be less by 16. What is the present price of an orange?
A. $2^{1}/_{2}$K B. $3^{1}/_{2}$K C. $5^{1}/_{2}$K D. 20K
E. $21^{1}/_{2}$K

33. A right circular cone has a base radius r cm and a vertical angle $2y°$. The height of the cone is
A. $r \tan y°$ cm   B. $r \sin y°$ cm
C. $r \cot y°$ cm   D. $r \cos y°$ cm
E. $r \csc y°$ cm

34. Omotola is in a plane at a height of 20,000 feet. She notices that an Island straight ahead is at an angle of depression of $4°$. Two minutes later, the angle of depression has grown to $6.4°$. Find the speed of the plane.
A. 375mph B. 480mph C. 512mph
D. 612mph E. 624mph

35. The formular $C = 1.5+0.5n$ gives the cost C (in naira) of feeding n poeple for a week. Find in kobo the extra cost of feeding one additional person.
A. 350k B. 200k C.150k D.50k E.45k

36. The perimeter of a rectangle is 38m. The difference between the length and the breadth is 3m.
What is the area of the rectangle?
A. 64m²   B. 78m²   C. 82m²
D. 88m²   E. 96m²

37. In a family of 21 poeple, the average age is 14years. If the age of the grandfahter is not counted, the average drops to 12years. What is the age of the grandfather?
A. 65 years B. 80 years C. 72 years
D. 54years E. 58 years

38. If the function f is defined by
$f(x+2) = 2x^2+7x-5$, find $f(-1)$
A. -10 B. -8 C. 4 D. 10 E. 8

39. In a geometrical progression, the Fourth term exceeds the third term by 72 and the third term exceeds the second term by 24. What is the frist term?
A. 1/4 B. 1/3 C. 2. D. 3 E. 4

40. The angle between the positive horizontal axis and a given line is $135°$. Find the equation of the line if it passes through the points (2,3)
A. $x-y=1$ B. $x+y=1$
C. $x+y=5$ D. $x-y=5$ E. $x+y=-5$

41. Evaluate x, if
$$\frac{\log_2 16 - \log_2 (1/4)}{\log_2 x} = 1$$
A. 6 B. 18 C. 32 D. 64 E. 128

42. If $a+b = 90°$,
simplify $(\sin a + \sin b)^2 - 2\sin a \sin b$
A. 0 B.1 C.45 D.90 E. 180

43. What is the probability that 3 customers waiting in a bank will be served in the sequence of their arrival at the bank?
A. 1 B.1/2 C.1/3 D.1/5 E. 1/6

44. When three is added to three fifths of a number, the result is the same as three-fourths of the number. Find the number.
A. 15 B. 16 C. 20 D. 25 E. 30

45. Which of the following is not a perfect square?
A. $x^2 + 4x + 16$ B. $x^2 + 6x + 9$
C. $4x^2 + 4x+1$ D. $1-2x+ x^2$ E. $9x^2 - 6x+1$

46. Find the mass of a cylindrical iron pipe 2m long and 6cm in external diameter if the metal is 0.5cm thick and its density is $78g/cm^3$
A. 9.1kg   B. 9.8kg   C. 10.6kg
D. 11.9kg   E. 12.1kg

47. A prism is 1m long and has a cross-section which is an isosceles triangle of sides 50cm, 50cm and 60cm. Calculate its total surface area in cm²
A 16000 B. 16200 C. 17200 D. 17500 E. 18400

48. A three digit number is formed by arranging 3, 2,8 in a random order. What is the probability that it is divisible by 4?
A. 1/2 B. 1/3 C. 1/4 D. 1/6 E. 2/3

49. If $\sin x = \cos x$. Find the value of x if it lies in the first quadrant.
A. 30° B. 35° C. 40° D. 45° E. 90°

50. Using the information in question 49 above, find the value of x if it lies in the third quadrant.
A. 180° B. 215° C. 225° D. 256° E. 360°