ABOUT THE BOOK

The title of this book is GEEPEE DRIVE, MTH 102. The book contains University 100 level Mathematics past questions and detailed solutions. The solutions are self-explanatory. The past questions are questions from Obafemi Awolowo University, Ile-Ife, Osun State, in Nigeria.

The topics covered in this book include:

1. Trigonometry
2. Differential Calculus
3. Integral Calculus
4. Differential Equations
5. Coordinate Geometry
6. Descriptive Statistics.

The book is very useful to the fresh undergraduates in 100 level studying Engineerings and Physical Sciences in any University, Polytechnic or any Higher Institution of learning. It is also very useful to students who are preparing for A Level Mathematics. Also, Further Maths students of WAEC, NECO, IGCSE and other similar examinations will find the book very useful.

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CHAPTER ONE

TRIGONOMETRY

Note the following important results in this topic:

1. $\sin^2 x + \cos^2 x = 1$ $\tan x = \frac{\sin x}{\cos x}$, $\cot x = \frac{\cos x}{\sin x}$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \text{cosec}^2 x$

2. **Compound Angles**

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

3. **Double Angles**

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

4. **Half Angles**

$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$

$\cos x = \cos^2\frac{x}{2} \sin^2\frac{x}{2}$

$= 2\cos^2\frac{x}{2} - 1$

$= 1 - 2\sin^2\frac{x}{2}$

$$\tan x = \frac{2\tan\frac{x}{2}}{1 - \tan^2\frac{x}{2}}$$

5. **Sum of Compound Angles**

From (2) above,

$\sin(A+B) + \sin(A - B) = 2\sin A\cos B$

$\sin(A+B) - \sin(A - B) = 2\cos A\sin B$

$\cos(A+B) + \cos(A - B) = 2\cos A\cos B$

$\cos(A+B) - \cos(A - B) = -2\sin A\sin B$

6. From (5) above, suppose $A + B = X$, and $A - B = Y$,

$A + B = X$

$\underline{A - B = Y}$ (adding)

$2A = X + Y$ $\quad\quad \therefore A = \frac{X+Y}{2}$

$A + B = X$

$\underline{A - B = Y}$ (subtracting)

$2B = X - Y$ $\quad\quad \therefore B = \frac{X-Y}{2}$

Hence,

Factor Formulas

$\sin X + \sin Y = 2\sin(\frac{X+Y}{2})\cos(\frac{X-Y}{2})$

$\sin X - \sin Y = 2\cos(\frac{X+Y}{2})\sin(\frac{X-Y}{2})$

$\cos X + \cos Y = 2\cos(\frac{X+Y}{2})\cos(\frac{X-Y}{2})$

$\cos X - \cos Y = -2\sin(\frac{X+Y}{2})\sin(\frac{X-Y}{2})$

7. **Negative Angles**

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$\tan(-x) = -\tan x$

8.

S (Sine positive) 2^{nd} Quadrant	**A** (All positive) 1^{st} Quadrant
T (Tan positive) 3^{rd} Quadrant	**C** (Cos positive) 3^{rd} Quadrant

9. **Sine and Cosine in terms of Double Angles**

From (3) above, i.e. $\cos 2x = 2\cos^2 x - 1$; $\cos 2x = 1 - 2\sin^2 x$

$$\cos x = \sqrt{\frac{1 + \cos 2x}{2}} \qquad \sin x = \sqrt{\frac{1 - \cos 2x}{2}}$$

10. **t - Formulas**

If $\tan \frac{x}{2} = t$, then,

$$\sin x = \frac{2t}{1 + t^2}, \quad \cos x = \frac{1 - t^2}{1 + t^2} \quad ; \quad \tan x = \frac{2t}{1 - t^2}$$

11. $a \sin\theta + b \cos\theta = R \sin(\theta + \alpha)$

$a \sin\theta - b \cos\theta = R\sin(\theta - \alpha)$

$a \cos\theta + b \sin\theta = R\cos(\theta - \alpha)$

$a \cos\theta - b \sin\theta = R\cos(\theta + \alpha)$

where $R = \sqrt{a^2 + b^2}$; $\tan \alpha = \frac{b}{a}$ $(0 < \alpha < 90^0)$.

12. **Sine Rule**

In any triangle ABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

where R is the radius of the circumcircle of triangle ABC.

Cosine Rule

$$a^2 = b^2 + c^2 - 2bc \operatorname{Cos} A$$

Cosine is basically applicable under the following conditions:

(1) when two sides and included angle are given

(2) when all the three sides are given

13. If $\sin \theta = a$

$\theta = n180^0 + (-1)^n a$ where $a = \sin^{-1}(a)$

If $\cos \theta = b$,

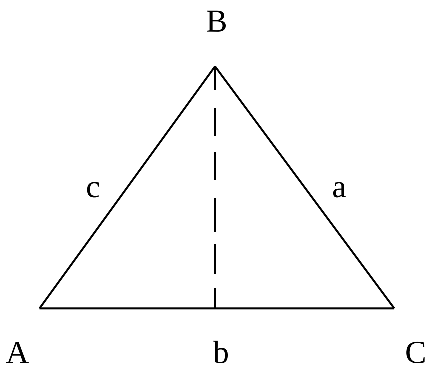
$\theta = n360^0 \pm \beta$ where $\beta = \cos^{-1}(b)$

If $\tan \theta = c$,

$\theta = n180^0 + \gamma$ where $\gamma = \tan^{-1}(c)$

Note that $n = 0, \pm 1, \pm 2, \ldots$..

14. Area of any triangle is given by:



Area = $\frac{1}{2}$ x base x height

= $\frac{1}{2}$ ab sin C

= $\frac{1}{2}$ ac sin B

= $\frac{1}{2}$ bc sin A

Hero's Formula

Area $= \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{a+b+c}{2}$

QUESTION 1

(a) Show that $2\cot^{-1}x = \cot^{-1}\left[\dfrac{x^2 - 1}{2x}\right]$.

(b) Given that $3\cos x = -5 - 2\sec x$, find all possible values of $\cos x$ and $\tan^2 x$.

Solution:

(a) Let $\cot^{-1}x = A$ and $\cot^{-1}\left[\dfrac{x^2 - 1}{2x}\right] = B$. So, $2A = B$.

When $\cot^{-1}x = A$, then, $\cot A = x \longrightarrow$

$\dfrac{1}{\tan A} = x$, $\tan A = \dfrac{1}{x}$.

Also, when $\cot^{-1}\left[\dfrac{x^2 - 1}{2x}\right] = B$,

then, $\cot B = \dfrac{x^2-1}{2x} \longrightarrow \dfrac{1}{\tan B} = \dfrac{x^2 - 1}{2x}$

$\therefore \tan B = \dfrac{2x}{x^2 - 1}$.

Remember $2A = B$. Taking tan of both sides:

$\tan 2A = \tan B$, $\tan B = \dfrac{2x}{x^2 - 1}$ ……………………………………RHS

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A} = \dfrac{2(1/x)}{1 - (1/x)^2} = \dfrac{(2/x)}{1 - (1/x)^2}$

$= \dfrac{2}{x} \div (1 - 1/x^2) = \left[\dfrac{2}{x} \div \dfrac{x2-1}{x}\right]$

$= \dfrac{2}{x} \times \dfrac{x^2}{x^2 - 1} = \dfrac{2x}{x^2 - 1}$ ……………………………………………LHS

$\therefore$ Since **LHS = RHS**, $2\cot^{-1}x = \cot^{-1}\left[\dfrac{x^2 - 1}{2x}\right]$

(b) $3\cos x = -5 - 2\sec x \implies 3\cos x + 5 + 2\sec x = 0$

$\implies 3\cos x + 5 + 2\left[\dfrac{1}{\cos x}\right] = 0$. Let $\cos x$ be P. $\longrightarrow$

$3P + 5 + \dfrac{2}{P} = 0$, $\longrightarrow 3P^2 + 5P + 2 = 0$

$3P^2 + 3P + 2P + 2 = 0$, $\longrightarrow 3P(P - 1) + 2(P + 1) = 0$,

$(P+1)(3P + 2) = 0$ $\therefore P = -1$ or $^{-2}/_3$

$\therefore \cos x = \underline{\underline{-1}}$ or $\underline{\underline{^{-2}/_3}}$.

To find the possible values of $\tan^2 x$, let's use the relationship:

$1 + \tan^2 x = \sec^2 x \longrightarrow \tan^2 x = \sec^2 x - 1$.

When $\cos x = -1$, the given equation $3\cos x = -5 - 2\sec x$ becomes: $3(-1) = -5 - 2\sec x$,

$-3 = -5 - 2\sec x$, $2\sec x = -5 + 3$, $2\sec x = -2$ $\therefore \sec x = -1$

$\therefore \sec^2 x = (-1)^2 = 1$. Hence, $\tan^2 x = \sec^2 x - 1 \longrightarrow \tan^2 x = 1 - 1 = 0$.

When $\cos x = {}^{-2}/_3$, the given equation becomes:

$3({}^{-2}/_3) = -5 - 2\sec x$, $-2 = -5 - 2\sec x$,

$\therefore \sec^2 x = ({}^{-3}/_2)^2 = {}^9/_4$. Hence, $\tan^2 x = \sec^2 x - 1$

$\Longrightarrow \tan^2 x = {}^9/_4 - 1 = {}^5/_4$

$\therefore$ The possible values of tan x = $\underline{0 \text{ or } {}^5/_4}$

QUESTION 2

(a) (i) Write down the expansions for $\sin(A + B)$ and $\cos(A + B)$ where A and B are angles of any magnitude.

(ii) Deduce that $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

(iii) $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

(iv) Hence or otherwise, show, that $\tan \frac{\pi}{12} = \left(\frac{1}{2+\sqrt{3}}\right)$.

(b) If A, B and C are the angles of triangle, prove that

$\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

Solution:

(a) (i) The expansions for $\sin(A+B)$ and $\cos(A+B)$ are:

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

(ii) $\tan(A+B) = \dfrac{\sin(A+B)}{\cos(A+B)} = \dfrac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B}$

Dividing both the numerator denominator by cosAcosB:

$$\left(\frac{\dfrac{\sin A\cos B + \cos A\sin B}{(\cos A\cos B)}}{\dfrac{\cos A\cos B - \sin A\sin B}{(\cos A\cos B)}}\right) \longrightarrow$$

$$\frac{\frac{\sin A\cos B}{\cos A\cos B} + \frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} - \frac{\sin A\sin B}{\cos A\cos B}} = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

(iii) From the above, $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

So,

$$\tan 2A = \tan(A+A) = \frac{\sin(A+A)}{\cos(A+A)} = \frac{taA + \tan A}{1- \tan A\tan A}$$

$$\therefore \tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

(iii) When you are given this type of question

i.e $\tan\frac{\pi}{12} = \frac{1}{2+\sqrt{3}}$, remember to always change the angle from radian to degree to make the manipulations easier.

So, remember 2π radians $= 360^0$

1 radian $= \frac{360}{2\pi}$

$\therefore \frac{\pi}{12}$ radians $= \frac{360}{2\pi} \times \frac{\pi}{12} = 15^0$

$\tan\frac{\pi}{12} = \tan 15^0$:

From the above, we have that:

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}.$$

If $A = 15$, then, $2A = 2 \times 15 = 30^0$.

So, $\tan 30^0 = \frac{2\tan 15^0}{1 - \tan^2 15^0}$

Let $\tan 15^0$ be t . So, $\tan 30^0 = \frac{2t}{1-t2}$

(Remember $\tan 30^0 = \frac{1}{\sqrt{3}}$)

So, $\frac{1}{\sqrt{3}} = \frac{2t}{1-t2}$, $1 - t^2 = 2\sqrt{3}\,t$, $\longrightarrow$ $t^2 + 2\sqrt{3}\,t - 1 = 0$,

This is a quadratic equation.

So, $t = \frac{-2\sqrt{3} \pm \sqrt{(2\sqrt{3})^2 - 4(1)(-1)}}{2(1)}$

$\therefore t = -\sqrt{3} + 2$ or $\sqrt{3} - 2$

Now taking the first value alone. i.e $-\sqrt{3} + 2$.

We have $-\sqrt{3} + 2 = 2 - \sqrt{3} = (2 - \sqrt{3}) \times \frac{(2 + \sqrt{3})}{(2 + \sqrt{3})}$

Note that $\frac{(2+\sqrt{3})}{(2+\sqrt{3})} = 1$. So, $\frac{(2-\sqrt{3}) \times (2+\sqrt{3})}{(2+\sqrt{3})}$

$$= \frac{2^2 - (\sqrt{3})^2}{(2+\sqrt{3})} = \frac{4-3}{(2+\sqrt{3})} = \frac{1}{(2+\sqrt{3})}$$

If you want to answer the question otherwise, you may take this option:

$\tan\frac{\pi}{12} = \tan 15^0 = \tan(45^0 - 30^0)$

$$\tan(45^0 - 30^0) = \frac{\tan 45^0 - \tan 30^0}{1 + \tan 45^0 \tan 30^0}$$

Remember $\tan 45^0 = 1$, $\tan 30^0 = \frac{1}{\sqrt{3}}$

So, $\tan(45^0 - 30^0) = \frac{1 - {}^1/_3}{\{1 + (1)({}^1/_3)\}}$

$$= \left(1 - \frac{1}{\sqrt{3}}\right) \div \left(1 + \frac{1}{\sqrt{3}}\right) = \left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) \div \left(\frac{\sqrt{3}+1}{\sqrt{3}}\right) = \left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) \times \left(\frac{\sqrt{3}}{\sqrt{3}+1}\right) = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

Rationalizing the denominator gives: $\frac{(\sqrt{3}-1)}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)} = \frac{3-\sqrt{3}-\sqrt{3}+1}{3-1} = \frac{4-2\sqrt{3}}{3-1} = \frac{4-2\sqrt{3}}{2} =$

$$2-\sqrt{3} = (2-\sqrt{3}) \times \frac{(2+\sqrt{3})}{(2+\sqrt{3})} = \frac{4-3}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}}.$$

(b) If A, B, and C are the angles of a triangle, then $A + B + C = 180^0$. Now, from the LHS of the given equation,

$\sin A + \sin B + \sin C = \sin A + \sin B + \sin[180^0 - (A+B)]$

{Note: $c = 180^0 - (A + B)$}

$\sin[(180^0 - (A+B)] = \sin 180^0 \cos(A + B) - \cos 180^0 \sin(A+B)$

$= 0 - (-1)\sin(A + B) = \sin(A+B)$

So, $\sin A + \sin B + \sin[180^0 - (A+B)] =$

$\sin A + \sin B + \sin(A+B) = (\sin A + \sin B) + \sin(A+B)$

$$= 2\sin\left[\frac{(A+B)}{2}\right]\cos\left[\frac{(A-B)}{2}\right] + \sin(A+B)$$

$$= 2\sin\left[\frac{(A+B)}{2}\right]\cos\left[\frac{(A-B)}{2}\right] + 2\sin\left[\frac{(A+B)}{2}\right]\cos\left[\frac{(A+B)}{2}\right]$$

$= 2\sin(\frac{A+B}{2})\left[\cos(\frac{A-B}{2}) + \cos(\frac{A+B}{2})\right]$

Let $(\frac{A-B}{2})$ be x and $(\frac{A+B}{2})$ be y .

So, we have:

$2\sin y\,(\cos x + \cos y)$

Remember $\cos x + \cos y = 2\cos(\frac{x+y}{2})\cos(\frac{x-y}{2})$

So, we now have:

$2\sin y\,(\cos x + \cos y) = (2\sin y)\{2\cos(\frac{x+y}{2})\cos(\frac{x-y}{2})\} = 4\sin y\cos(\frac{x+y}{2})\cos(\frac{x-y}{2})\} =$

$4\sin(\frac{A+B}{2})\cos(\frac{A}{2})\cos(\frac{-B}{2})$ { $\frac{A}{2}$ is got from $\frac{x+y}{2}$ i.e $[\frac{A-B}{2} + \frac{A+B}{2}] \div 2$. Same with $\frac{-B}{2}$ }.

$= 4\sin(\frac{A+B}{2})\cos(\frac{A}{2})\cos(\frac{B}{2})$ $\{\cos(-\theta) = \cos\theta\}$

From $A + B + C = 180$, $A + B = 180^0 - C$. So, $\frac{A+B}{2} = \frac{180-C}{2} = 90^0 - \frac{C}{2}$

So, $\sin(\frac{A+B}{2}) = \sin(90^0 - \frac{C}{2}) = \sin 90^0\cos(\frac{C}{2}) - \cos 90^0\sin(\frac{C}{2}) = \cos(\frac{C}{2}) - 0 = \cos(\frac{C}{2})$

So, $4\sin(\frac{A+B}{2})\cos(\frac{A}{2})\cos(\frac{B}{2}) = 4\cos(\frac{C}{2})\cos(\frac{A}{2})\cos(\frac{B}{2}) = 4\cos(\frac{A}{2})\cos(\frac{B}{2})\cos(\frac{C}{2})$

Note: If A, B, and C are the angles of a triangle:

$A + B + C = 180^0$

$\sin C = \sin(A + B)$,

$\cos C = -\cos(A + B)$

$\tan C = -\tan(A + B)$.

<u>QUESTION 3</u>

(a) (i) If $\tan(\frac{\theta}{2}) = 1$, prove that $\sin\theta = \frac{2t}{1 - t^2}$ and $\cos\theta = \frac{(1 - t^2)}{(1 + t^2)}$.

(ii) Hence or otherwise, solve the trigonometric equation:

$3\sin\theta + 4\cos\theta = 5$ (in the range $0 \leq \Theta \leq 180^0$).

(b) (i) By using the formula $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC} = 2R$, prove that $\tan(\frac{B-C}{2}) = \frac{b-c}{b+c}\cot\frac{A}{2}$.

(ii) Hence, solve the following triangle completely:

$b = 5.62$, $c = 4.13$ and $A = 62^0$.

Solution:

(a) (i) $\sin\theta = 2\sin(\frac{\theta}{2})\cos(\frac{\theta}{2}) = \dfrac{2\sin(\frac{\theta}{2})\cos(\frac{\theta}{2})}{1}$

Dividing the numerator and the denominator by $\cos^2(\frac{\theta}{2})$ gives:

$$\frac{\{2\sin(\frac{\theta}{2})\cos(\frac{\theta}{2})\}/\cos^2(\frac{\theta}{2})}{1/\cos^2(\frac{\theta}{2})}$$

$$= \frac{2\sin(\frac{\theta}{2})\cos(\frac{\theta}{2})}{\cos^2(\frac{\theta}{2})} \div \frac{1}{\cos^2(\frac{\theta}{2})} = \frac{2\sin(\frac{\theta}{2})}{\cos(\frac{\theta}{2})} \div \frac{1}{\cos^2(\frac{\theta}{2})} = 2\tan(\tfrac{\theta}{2}) \div \sec^2(\tfrac{\theta}{2})$$

$$= \frac{2\tan(\frac{\theta}{2})}{\sec^2(\frac{\theta}{2})} = \frac{2\tan(\frac{\theta}{2})}{1 + \tan^2(\frac{\theta}{2})}$$ (Remember $1 + \tan x = \sec^2 x$)

$$= \frac{2t}{1 - t^2}$$

To get the complete past questions and solutions/explanations on **Trigonometry**, you can contact: 08033487161, 08177093682 or osospecial2015@yahoo.com for just N500 ($1.25).

You can also get the past questions and solutions/explanations for the remaining topics on MTH 102. The remaining topics are:

- Trigonometry

- Differential Calculus

- Integral Calculus

- Differential Equations

- Coordinate Geometry

- Descriptive Statistics.