CAMBRIDGE INTERNATIONAL EXAMINATIONS

International General Certificate of Secondary Education

MARK SCHEME for the May/June 2014 series

0606 ADDITIONAL MATHEMATICS

0606/22 Paper 2, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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1	rationalise the denominator to get $\dfrac{\left(2+\sqrt{5}\right)^2\left(\sqrt{5}+1\right)}{5-1}$ or better	**M1**	or squaring to get $\dfrac{\left(4+4\sqrt{5}+5\right)}{\sqrt{5}-1}$ or better
	squaring to get $\dfrac{\left(4+4\sqrt{5}+5\right)\left(\sqrt{5}+1\right)}{their4}$ or better	**M1**	or rationalising the denominator to get $\dfrac{their\left(9+4\sqrt{5}\right)\left(\sqrt{5}+1\right)}{5-1}$ or better
	$\dfrac{29}{4}+\dfrac{13}{4}\sqrt{5}$ oe isw	**A1 + A1**	correct simplification Allow $\dfrac{29+13\sqrt{5}}{4}$ etc.
2	Correctly eliminate y	**M1**	$-kx+2=2x^2-9x+4$ oe
	$2x^2+(k-9)x+2[=0]$ oe	**A1**	allow even if x terms not collected; condone … $= y$ provided later work implies it should be 0
	Use b^2-4ac oe	**M1**	must be applied to a 3 term quadratic expression containing k as a coefficient; condone < 0 etc.
	Reach $their(k-9=\pm4)$ or solves $their\left(k^2-18k+65\right)=0$	**M1**	condone $9-k=\pm4$; condone an inequality at this stage
	$k=5$ and 13 cao	**A1**	mark final answer, do not isw; **A0** if inequalities for final answers

3 **(i)**	$3(-1)^3 - 14(-1)^2 - 7(-1) + d = 0$ with completion to $d = 10$	**B1**	at least $-3 - 14 + 7 + d = 0$, $d = 10$; N.B. = 0 must be seen or implied by … = d or … = $-d$, may be seen in following step. or convincingly showing $3(-1)^3 - 14(-1)^2 - 7(-1) + 10 = 0$; at least $-3 - 14 + 7 + 10 = 0$ or correct synthetic division at least as far as $\begin{array}{r\|rrrr} -1 & 3 & -14 & -7 & 10 \\ & & -3 & 17 & -10 \\ \hline & 3 & -17 & 10 & \end{array}$
(ii)	$3x^2 - 17x + 10$ isw or $a = 3$, $b = -17$, $c = 10$ isw	**B2, 1, 0**	-1 each error; must be seen or referenced in **(ii)** even if found in **(i)** or **(iii)**
(iii)	$(x+1)(x-5)(3x-2)$	**M1**	for factorising quadratic **ft** correct; condone omission of $(x+1)$ or for **ft** correct use of formula or **ft** correct completing the square
	$-1, 5, \frac{2}{3}$	**A1**	If **M0** then **SC1** for all three roots stated without working or verified/found by trials

4 **(i)**	$12\left(x-\frac{1}{4}\right)^2+\frac{17}{4}$ isw	**B3, 2, 1,0**	one mark for each of p, q, r correct in a correctly formatted expression; allow correct equivalent values; If **B0** then **SC2** for $12\left(x-\frac{1}{4}\right)+\frac{17}{4}$ or **SC1** for correct 3 values seen in incorrect format e.g. $12\left(x-\frac{1}{4}x\right)+\frac{17}{4}$ or $12\left(x^2-\frac{1}{4}\right)+\frac{17}{4}$ or for a correct completed square form of the original expression in a different but correct format. e.g. $3\left(2x-\frac{1}{2}\right)^2+\frac{17}{4}$
(ii)	*their* $\frac{4}{17}$ or *their* 0.235	**B1ft**	strict **ft** ; *their* $\frac{4}{17}$ must be a proper fraction or decimal rounded to 3sig figs or more or truncated to 4 figs or more
	their $x=\frac{1}{4}$ oe	**B1ft**	strict **ft** ; x must be correctly attributed
5 **(i)**	$1-20x+160x^2$	**B2, 1, 0**	−1 each error if **B0** then **M1** for 3 correct terms seen; may be unsimplified e.g. $1,\ 5(-4x),\ \frac{5\times 4}{2}(-4x)^2$
(ii)	$a+(their\ -20)=-23$ soi	**M1**	condone sign errors only; must be *their* −20 from **(i)**
	$a=-3$	**A1**	validly obtained
	$b+(their\ -20)a+(their 160)=222$ soi	**M1**	condone sign errors only ; must be *their* −20 and *their*160 from **(i)** and *their a* if used
	$b=2$	**A1**	validly obtained

6 (a) (i)	1	**B1**	
(ii)	$x = -1$ or -2	**B1 + B1**	as final answers
(b)	$\frac{\log_3 5}{\log_3 a}$ seen or implied	**B1***	may be implied by $2\log_3 15 - \log_3 5$
	$2\log_3 15 = \log_3 15^2$ seen or implied	**B1**	
	$\log_3 15^2 - \log_3 5 = \log_3\left(\frac{15^2}{5}\right)$	**B1dep***	not from wrong working
	$\log_3 45$ cao	**B1**	must be 45 not e.g. $\frac{225}{5}$; with no wrong working seen
7 (i)	$x^4\left(3e^{3x}\right) + 4x^3e^{3x}$ isw	**B1 + B1**	each term of the **sum** correct; must be a sum of two terms
(ii)	$\frac{1}{2+\cos x}\times(-\sin x)$ isw	**B2**	or **B1** for $\frac{1}{2+\cos x}\times(k \pm \sin x)$ and k a constant
(iii)	$\frac{d}{dx}(\sin x) = \cos x$ soi	**B1**	
	$\frac{d}{dx}\left(1+\sqrt{x}\right) = \frac{1}{2}x^{-\frac{1}{2}}$ soi	**B1**	
	$\frac{\left(1+\sqrt{x}\right) \textit{their} \cos x - \left(\textit{their}\frac{1}{2}x^{-\frac{1}{2}}\right)\sin x}{\left(1+\sqrt{x}\right)^2}$ isw	**B1ft**	for correct form of quotient rule **ft** their $\cos x$ and their $\frac{1}{2}x^{-\frac{1}{2}}$; allow correct use of product and chain rules to obtain $\sin x\left(-\left(1+\sqrt{x}\right)^{-2}\times\frac{1}{2}x^{\frac{1}{2}}\right) +$ $\cos x\left(1+\sqrt{x}\right)^{-1}$ oe

8	Substitution of either $x-5$ or $y+5$ into equation of curve and brackets expanded	**M1**	condone one sign error in either equation of curve or expansion of brackets; condone omission of = 0, BUT $x-5$ or $y+5$ must be correct
	$2x^2-8x-10[=0]$ or $2y^2+12y[=0]$ obtained	**A1**	
	Solving their quadratic	**M1**	dep on a valid substitution attempt
	$(-1, -6)$ oe and $(5, 0)$ oe isw	**A1*+A1***	or **A1** for correct pair of x coordinates or correct pair of y coordinates
	$\sqrt{72}$ or $6\sqrt{2}$ cao isw	**B1dep***	
9 (i)	$[y=]\dfrac{(2x+1)^{\frac{3}{2}}}{2\times\frac{3}{2}}(+c)$ oe	**B2**	or **B1** for $(2x+1)^{\frac{1}{2}+1}$
	$10=\dfrac{2}{6}(2(4)+1)^{\frac{3}{2}}+c$ oe	**M1**	for valid attempt to find c; condone slips e.g. omission of power or sign error
	$y=\dfrac{(2x+1)^{\frac{3}{2}}}{2\times\frac{3}{2}}+c$ seen and $c=1$ or $y=\dfrac{(2x+1)^{\frac{3}{2}}}{2\times\frac{3}{2}}+1$ isw	**A1**	must have $y=$ ….; condone $f(x)=$ ...
(ii)	$\int\left(\frac{1}{3}(2x+1)^{\frac{3}{2}}+1\right)dx=\frac{1}{15}(2x+1)^{\frac{5}{2}}+x(+const)$	**B1 + B1**	**B1** for $(2x+1)^{\frac{3}{2}+1}$, **B1** for $\frac{1}{15}(2x+1)^{\frac{5}{2}}$
		B1ft	**B1 ft** *their c* from **(i)** provided $c\neq 0$
	$\left[\frac{1}{15}(2x+1)^{\frac{5}{2}}+x\right]_0^{1.5}=$ $\left[\frac{1}{15}(2(1.5)+1)^{\frac{5}{2}}+(1.5)\right]-\left[\frac{1}{15}(2(0)+1)^{\frac{5}{2}}+0\right]$	**M1**	for a genuine attempt to find F(1.5) – F(0) in an attempt to integrate *their y*; if their F(0) is 0 must see at least their F(1.5)– 0; condone + c as long as their c is **not** numerical.
	$\dfrac{107}{30}$ oe isw	**A1**	if decimal 3.57 or more accurate e.g. 3.566

10 (i)	Taking logs of both sides	**M1**	any base; must be an explicitly correct statement
	$\log y = \log A + x \log b$	**A1**	correct form; any base; no recovery from incorrect method steps
(ii)	b: awrt 3 to one sf isw or awrt 4 to one sf isw	**B2**	or **M1** for $b = e^{\textit{their}\text{ gradient}}$ soi; their gradient must be correctly evaluated as rise/run
	A: awrt 0.5 to one sf	**B2**	or **B1** for $A = e^{-0.6}$ or **SC1** for $A = e^{-0.3} = 0.7$ (giving an awrt 0.7)
(iii)	Evidence of graph used at ln y = 5.4 soi	**M1**	or $\dfrac{220}{\textit{their}0.5} = (\textit{their}4)^x$ or 5.39…= *their*(1.4)x + *their* –0.6 or $\ln(220) = x\ln(\textit{their}4) + \ln(\textit{their}0.5)$
	awrt 4.4 to two sf	**A1**	

11 (i)	$f(x) > 3$ or $[f(x) \in](3, \infty)$	**B1**	condone $y > 3$
(ii)	$x + 1 = 2^y$ $f^{-1}(x) = \log_2(x+1)$	**M1** **A1**	or $y + 1 = 2^x$ mark final answer or $\log_2(y+1) = x$ and $f^{-1}(x) = \log_2(x+1)$ or for $f^{-1}(x) = \dfrac{\log(x+1)}{\log 2}$ (any base for this form)
	Domain $x > 3$	**B1ft**	**ft** their **range** of f provided mathematically valid inequality or interval
	Range $f^{-1}(x) > 2$	**B1**	condone f(x) > 2 or y > 2
(iii)	$2^x(2^x - 1)$ oe isw	**B1**	e.g. $(2^x - 1)^2 + (2x - 1)$ or $2^{2x} - 2 \times 2^x + 1 + 2^x - 1$
	$2^x(2^x - 1) = 0$ **leading to** $2^x = 0$, impossible oe	**B1**	or $2^x = 0$ which is outside domain of gf
	$2^x = 1 \Rightarrow x = 0$	**M1**	or $2^x(2^x - 1) = 2^{2x} - 2^x = 0$ $[2^{2x} = 2^x] \Rightarrow x = 0$
	0 is not in the domain (and so $gf(x) = 0$ has no solutions)	**A1**	

12 (i)	$\frac{dy}{dx} = 3x^2 - 18x + 24$	**B1**	
	Solving their $3x^2 - 18x + 24 \geqslant 0$ by factorising or quadratic formula or completing the square	**M1**	attempt at differentiation resulting in quadratic expression with two terms correct; allow = or $\leqslant$ or < or > or $\geqslant$ 0 omitted here.
	Critical values 2 and 4	**A1**	
	$x \leqslant 2, x \geqslant 4$	**A1**	**A0** if spurious attempt to combine; mark final answer
(ii)	Evaluating their $\frac{dy}{dx}$ at $x = 3$	**M1**	
	Use of $m_1 m_2 = -1$ to get $m_{normal} = -\frac{1}{their(-3)}$	**M1**	must be explicit statement of gradient of normal ; may be seen in equation
	$y = 18$ soi	**B1**	
	$y - their18 = \left(their\frac{1}{3}\right)(x-3)$ or $y = their\frac{1}{3}x + c$ and $c = their17$ isw	**A1ft**	**ft** *their m* provided a genuine attempt at m_{normal} ; no **ft** if $m = their\ m_{\text{tangent}}$
	$P(0, 17)$ cao	**B1**	